# Truthful mechanisms for ownership transfer with expert advice

Ioannis Caragiannis, Aris Filos-Ratsikas, Swaprava Nath, and Alexandros A. Voudouris

## The setting

- One item for sale
- Two potential buyers $A$ and $B$ with monetary values $w_A$ and $w_B$ for the item
- One expert with vNM values $v(\cdot)$ for the two buyers and the option $\oslash$ of not selling

**Objectives**
- incentivize the buyers and the expert to truthfully report their preferences, and
- choose an option $o \in \{A, B, \oslash\}$ to maximize the social welfare

$$\mathrm{SW}(o) = \begin{cases} v(o) + \frac{w_o}{\max\{w_A, w_B\}}, & o \in \{A, B\} \\ v(\oslash), & \text{otherwise} \end{cases}$$

**Mechanism design**
- with money for the buyers
- without money for the expert

**Applications**
- Privatization of government assets
- Sports tournaments hosting

## Profile representations

- Two different views of a preference profile $\Pi$, depending on whether we sort the values in terms of the expert or the buyers

$$\Pi_E = \begin{pmatrix} 1 & x & 0 \\ h & \ell & z \end{pmatrix}$$

*Expert's view:* a mechanism is a lottery assigning probabilities $g(\Pi_E)$, $f(\Pi_E)$ and $\eta(\Pi_E)$ to the expert's first, second and third favorite option

$$\Pi_b = \begin{bmatrix} h & \ell & n \\ 1 & y & 0 \end{bmatrix}$$

*Buyers' view:* a mechanism is a lottery assigning probabilities $d(\Pi_b)$, $c(\Pi_b)$ and $e(\Pi_b)$ to the high-bidder, low-bidder and the option $\oslash$

## Truthfulness conditions

A mechanism is **truthful** if it is
- **Ech-IC**: the expert has no incentive to attempt any *level change in the reported valuation* which would change her second highest valuation;
- **ESw-IC:** the expert has no incentive to attempt a *reported valuation swap* which would change the order of her valuations for the options;
- **Bch-IC:** the buyers have no incentive to attempt *level changes in their reported bids* which would change their bids;
- **BSw-IC:** the buyers have no incentive to attempt *bid swaps* which would change the order of the bids

## Bch-IC characterization

**Lemma 1** (Myerson, 1981)**.** *A mechanism is BCh-IC iff the functions $d(\Pi_b)$ and $c(\Pi_b)$ are non-increasing and non-decreasing in terms of $y$, respectively.*

## Bch-IC characterization

**Lemma 2.** *A mechanism is ECh-IC iff the function $f(\Pi_E)$ is non-decreasing in $x$ and*

$$g(x, \cdot) = g(0, \cdot) - xf(x, \cdot) + \int_0^x f(t, \cdot)\,\mathrm{d}t,$$

## Overview of results

- We consider several classes of truthful mechanisms, depending on the level of information they use
- For each such class, we identify the best possible mechanism in terms of its approximation ratio with respect to the optimal social welfare

| class of mechanisms | apx. ratio |
|---|---|
| ordinal | 1.5 |
| bid-independent | 1.377 |
| expert-independent | 1.343 |
| randomized template | 1.25 |
| deterministic template | 1.618 |
| all deterministic | $\geq 1.618$ |
| all mechanisms | $\geq 1.14$ |

## Ordinal mechanisms

- Base the decision only on the relative order of the values reported by the expert or the buyers
- Mechanism **EOM:** select the expert's favorite option with probability $2/3$ and the expert's second favorite option with probability $1/3$
- Mechanism **BOM:** select the high-bidder with probability $2/3$ and the low-bidder with probability $1/3$

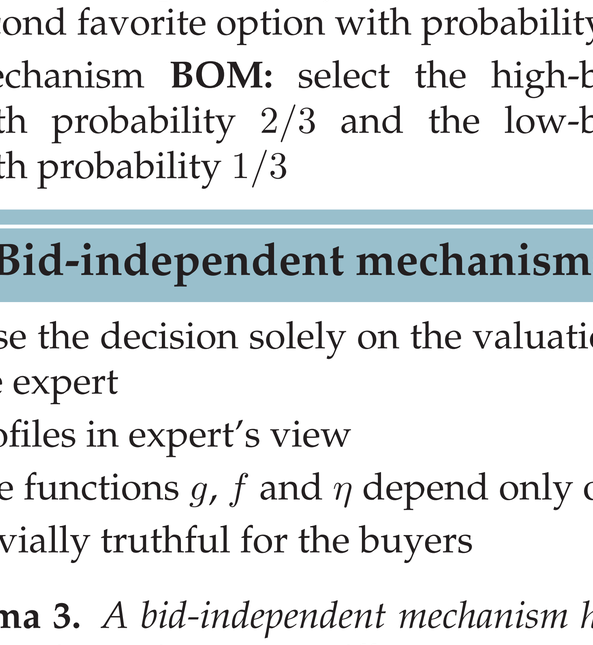
## Bid-independent mechanisms

- Base the decision solely on the valuations of the expert
- Profiles in expert's view
- The functions $g$, $f$ and $\eta$ depend only on $x$
- Trivially truthful for the buyers

**Lemma 3.** *A bid-independent mechanism has approximation ratio at most $\rho$ iff*

$$2g(x) + xf(x) \geqslant 2/\rho$$
$$g(x) + (1+x)f(x) \geqslant (1+x)/\rho.$$

**Lemma 4.** *An ECh-IC bid-independent mechanism is truthful iff $g(x) \geqslant f(x')$ and $f(x) \geqslant \eta(x')$ for every pair $x, x' \in (0, 1)$.*




Mechanism **BIM**

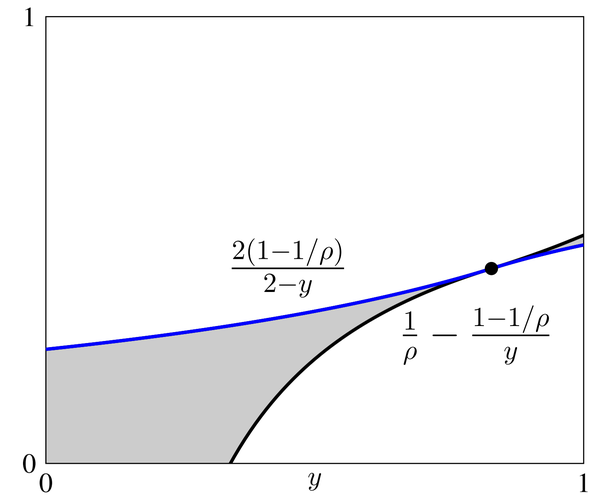
## Expert-independent mechanisms

- Base the decision solely on the bids of the buyers
- Profiles in buyers' view
- The function $c$ depends only on $y$; $d = 1 - c$ and $e = 0$
- Trivially truthful for the expert

**Lemma 5.** *An expert-independent mechanism has approximation ratio at most $\rho$ iff*

$$\frac{1}{\rho} - \frac{1 - 1/\rho}{y} \leqslant c(y) \leqslant \frac{2(1 - 1/\rho)}{2 - y}.$$

**Lemma 6.** *A BCh-IC expert-independent mechanism is truthful if and only if $d(1) \geqslant c(1)$.*



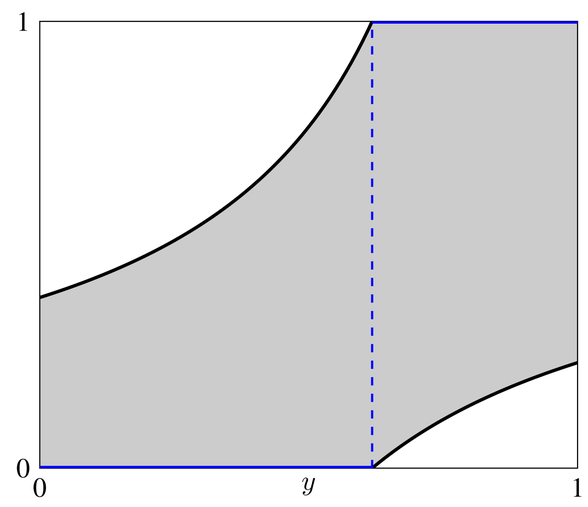

Mechanism **EIM**

## Template $\mathcal{T}$

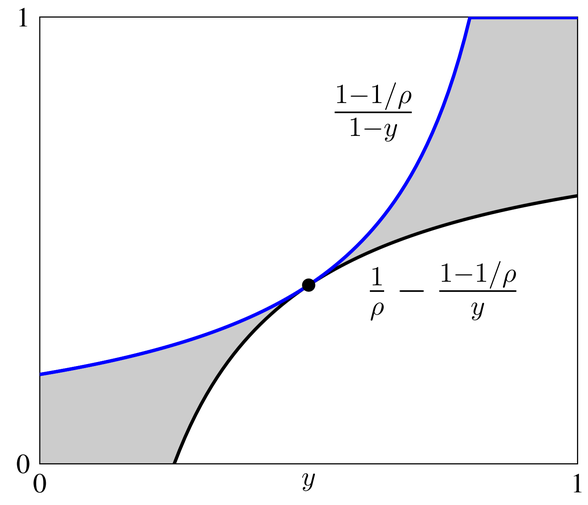
- Profiles in buyers' view with $\ell \geq h$ belong to category T1; all others belong to T2
- For every profile in T1, select the low-bidder with probability $c(y, T1)$; for every profile in T2, select the high-bidder with probability 1.

**Lemma 7.** *Any $\mathcal{T}$ mechanism is truthful and has approximation ratio at most $\rho$ iff*

$$\frac{1}{\rho} - \frac{1 - 1/\rho}{y} \leqslant c(y, T1) \leqslant \frac{1 - 1/\rho}{1 - y}.$$




Deterministic mechanism **D**




Randomized mechanism **R**