Truthful mechanisms for ownership transfer with expert advice
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The setting
- One item for sale
- Two potential buyers \( A \) and \( B \) with monetary values \( w_A \) and \( w_B \) for the item
- One expert with vNM values \( v(\cdot) \) for the two buyers and the option \( o \) of not selling

Objectives
- incentivize the buyers and the expert to truthfully report their preferences, and
- choose an option \( o \in \{ A, B, o \} \) to maximize the social welfare

\[
SW(o) = \begin{cases} 
 v(o) + \frac{w}{\max\{w_A, w_B\}}, & o \in \{A, B\} \\
 v(o), & \text{otherwise} 
\end{cases}
\]

Mechanism design
- with money for the buyers
- without money for the expert

Applications
- Privatization of government assets
- Sports tournaments hosting

Profile representations
- Two different views of a preference profile \( \Pi \), depending on whether we sort the values in terms of the expert or the buyers

\[
\Pi_E = \begin{pmatrix} 1 & x & 0 \\ h & \ell & z \end{pmatrix}
\]

Expert’s view: a mechanism is a lottery assigning probabilities \( g(\Pi_E), f(\Pi_E) \) and \( v(\Pi_E) \) to the expert’s first, second and third favorite option

\[
\Pi_b = \begin{pmatrix} h & \ell & n \\ 1 & y & 0 \end{pmatrix}
\]

Buyers’ view: a mechanism is a lottery assigning probabilities \( d(\Pi_b), c(\Pi_b) \) and \( v(\Pi_b) \) to the high-bidder, low-bidder and the option \( o \)

Truthfulness conditions
- A mechanism is truthful if it is
  - **ECh-IC**: the expert has no incentive to attempt any level change in the reported valuation which would change her second highest valuation;
  - **ESw-IC**: the expert has no incentive to attempt a reported valuation swap which would change the order of her valuations for the options;
  - **Bch-IC**: the buyers have no incentive to attempt level changes in their reported bids which would change their bids;
  - **BSw-IC**: the buyers have no incentive to attempt bid swaps which would change the order of the bids

Bch-IC characterization

**Lemma 1** (Myerson, 1981). A mechanism is BCh-IC if the functions \( d(\Pi_b) \) and \( c(\Pi_b) \) are non-increasing and non-decreasing in terms of \( y \), respectively.

**Lemma 2.** A mechanism is ECh-IC iff the function \( f(\Pi_E) \) is non-decreasing in \( x \) and

\[
g(x) = 0 \cdot - x f(x) + \int_0^x f(t) \cdot dt.
\]

Overview of results
- We consider several classes of truthful mechanisms, depending on the level of information they use
- For each such class, we identify the best possible mechanism in terms of its approximation ratio with respect to the optimal social welfare

<table>
<thead>
<tr>
<th>class of mechanisms</th>
<th>apx. ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>ordinal</td>
<td>1.5</td>
</tr>
<tr>
<td>bid-independent</td>
<td>1.377</td>
</tr>
<tr>
<td>expert-independent</td>
<td>1.343</td>
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<tr>
<td>randomized template</td>
<td>1.25</td>
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<tr>
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<tr>
<td>all deterministic</td>
<td>≥ 1.618</td>
</tr>
<tr>
<td>all mechanisms</td>
<td>≥ 1.14</td>
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</tbody>
</table>

Bid-independent mechanisms
- Base the decision solely on the valuations of the expert
- Profiles in expert’s view
- The functions \( g, f \) and \( \eta \) depend only on \( x \)
- Trivially truthful for the buyers

**Lemma 3.** A bid-independent mechanism has approximation ratio at most \( \rho \) ifff

\[
2g(x) + xf(x) \geq 2/\rho \\
g(x) + (1 + x)f(x) \geq (1 + x)/\rho.
\]

**Lemma 4.** An ECh-IC bid-independent mechanism is truthful ifff \( g(x) \geq f(x') \) and \( f(x) \geq \eta(x') \) for every pair \( x, x' \in (0, 1) \).

Expert-independent mechanisms
- Base the decision solely on the bids of the buyers
- Profiles in buyers’ view
- The function \( c \) depends only on \( y; d = 1 - c \) and \( e = 0 \)
- Trivially truthful for the expert

**Lemma 5.** An expert-independent mechanism has approximation ratio at most \( \rho \) ifff

\[
1 - 1/\rho \leq c(y) \leq 2(1 - 1/\rho)/2 - y.
\]

**Lemma 6.** A BCh-IC expert-independent mechanism is truthful if and only if \( d(1) \geq c(1) \).

Template \( T \)
- Profiles in buyers’ view with \( \ell \geq h \) belong to category \( T_1 \); all others belong to \( T_2 \)
- For every profile in \( T_1 \), select the low-bidder with probability \( c(y) \); for every profile in \( T_2 \), select the high-bidder with probability \( 1 \).

**Lemma 7.** Any \( T \) mechanism is truthful and has approximation ratio at most \( \rho \) ifff

\[
1 - 1/\rho \leq c(y, T_1) \leq 1 - 1/\rho - y.
\]