The Computational Power of Simple Protocols for Self-Awareness on Graphs

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Population Protocols

[AADFP, PODC '04]

- n mobile agents (finite-state)
- mobility
  - passive: uncontrollable by the agents
  - fair: protocols make progress
- rendezvous communication mechanism (ordered)
- no time-guarantee on causal influence: protocols can only stabilize
- Fairly weak model on complete graphs: semilinear predicates [AAE, PODC '06] even if memory becomes $o(\log \log n)$ [CMNPS, FOMC '11]

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Mediated Population Protocols

[CMS, ICALP '09]

- An additional limited mediator:
  - modeled by a constant memory between each pair of agents
  - e.g. a fixed meeting point for each pair
  - or a limited global storage
- Very strong model on complete graphs: symmetric predicates in **NSPACE**(n^2) [CMNPS, MFCS '10].

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Mediated Graph Protocols

MPPs and additionally:

- a complete communication graph $K_n = (V, E)$
- an input graph $G_\iota[K_n] = (V', E')$ specified by some network initialization function $\iota : V \cup E \to \{q_0, q_1, s_0, s_1\}$
- We are interested in protocols that (stably) decide properties of the input graph
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Our Fundamental Question

Can we characterize the class **GMGP** of decidable properties?

An Example: count-3-in-neighbors

Is there some node with ≥ 3 in-neighbors?

- $Q = \{q_0, q_1, q_2, q_3, q_4\}$
- $S = \{s_0, s_1\}$
- only q_4 outputs 1
- if $i < 4$ and $j < 3$ then $\delta(q_i, q_j, s_1) = (q_i, q_{j+1}, s_0)$
- if $i = 4$ or $j \geq 3$ then $\delta(q_i, q_j, s_1) = (q_4, q_4, s_0)$
- if $i = 4$ or $j = 4$ then $\delta(q_i, q_j, s_0) = (q_4, q_4, s_0)$

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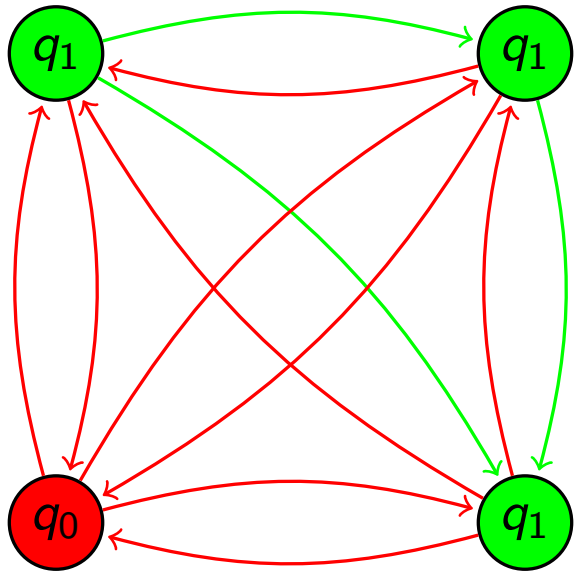

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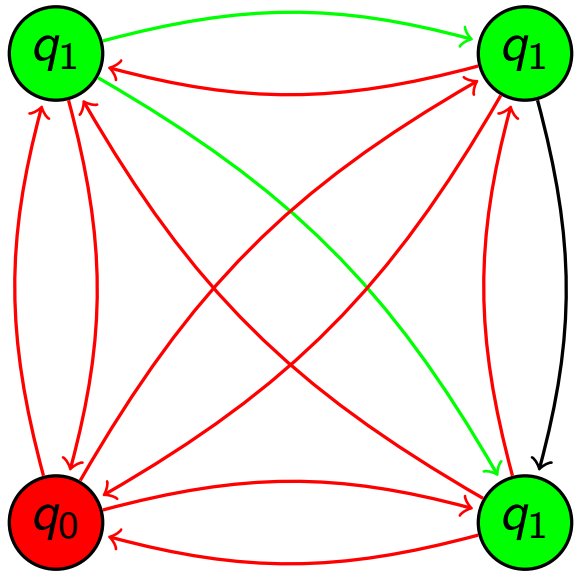

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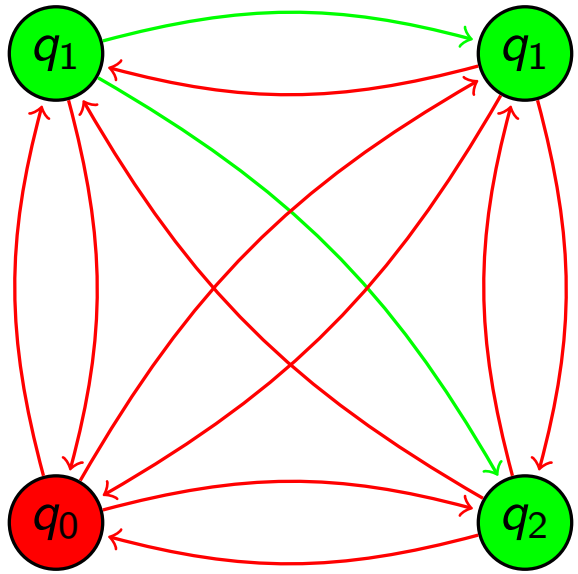

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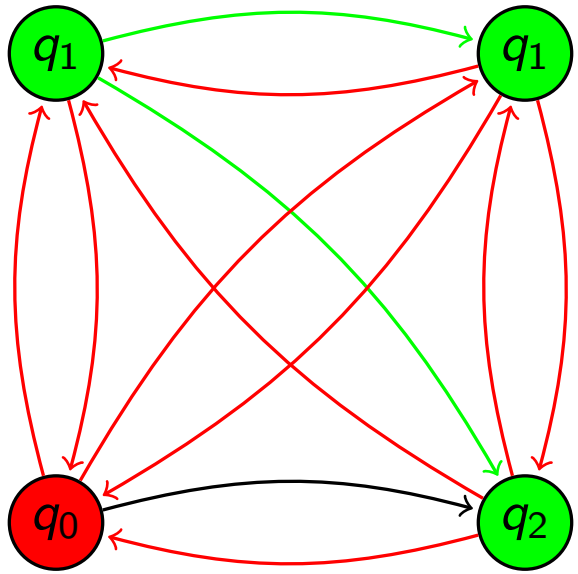

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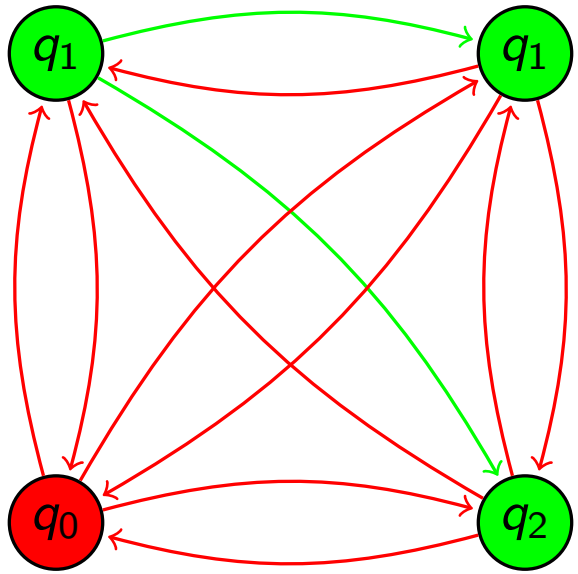

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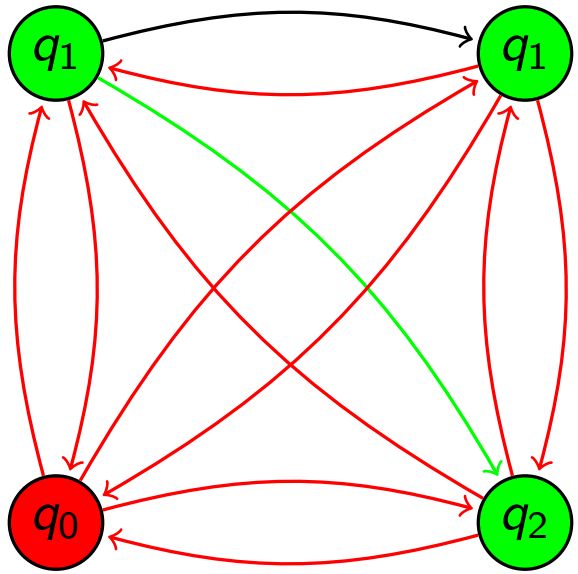

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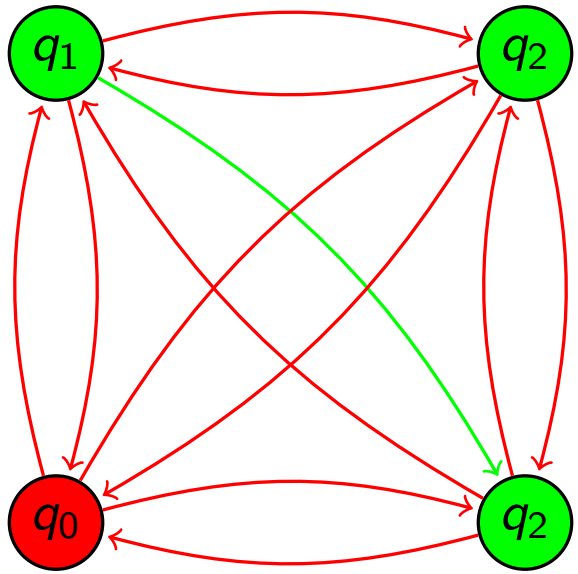

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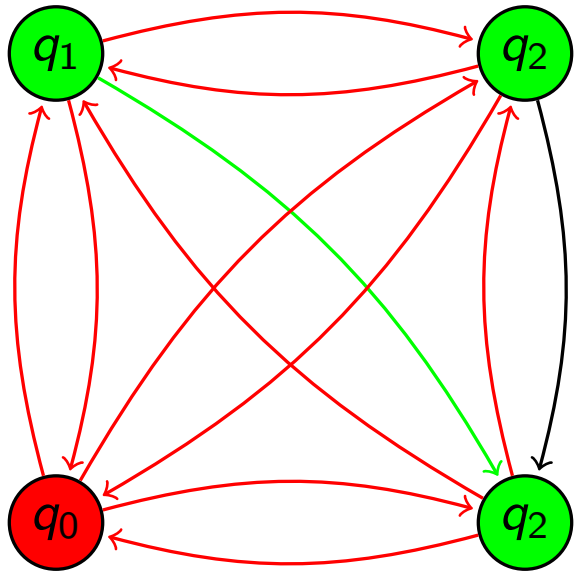

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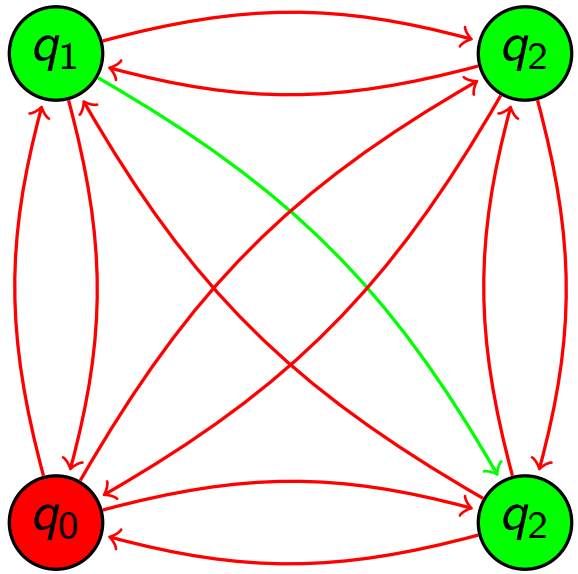

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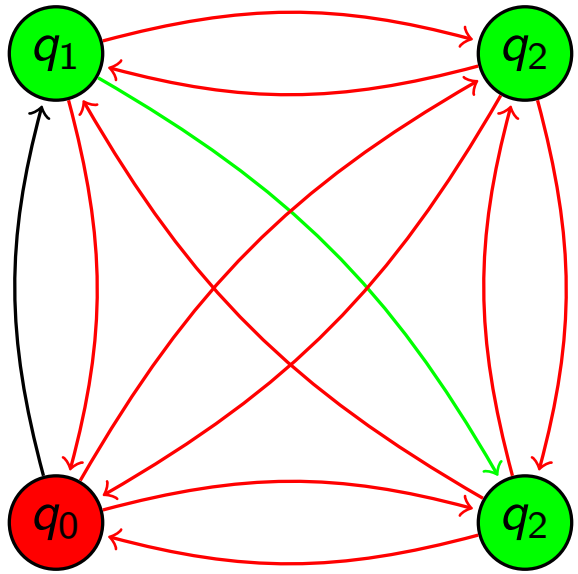

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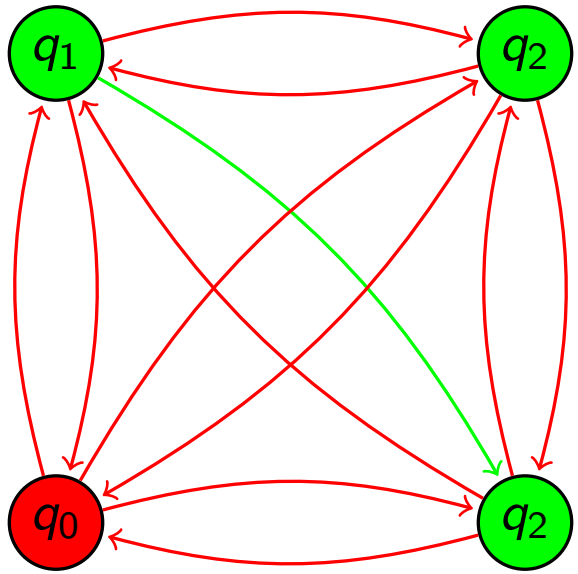

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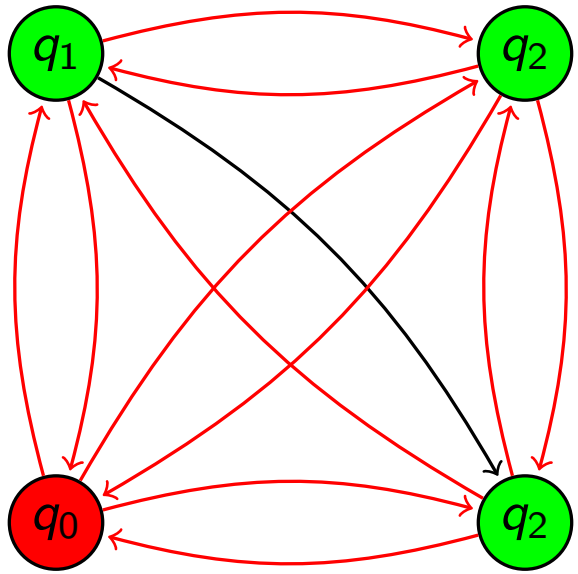

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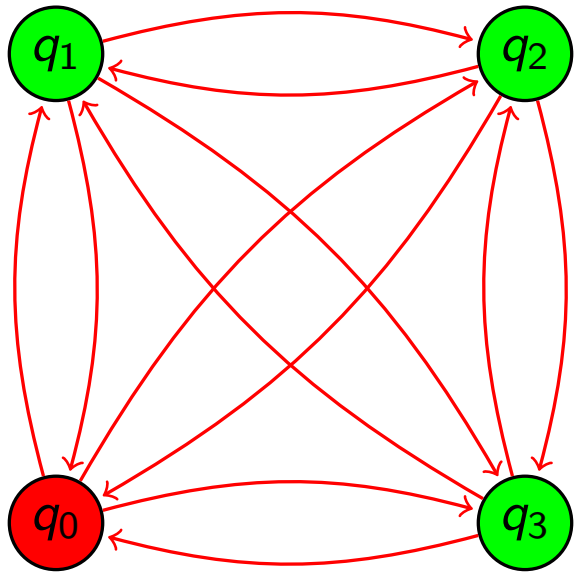

All agents forever output 0

Properties of MGPs

Theorem

GMGP *is closed under union, intersection, complement, and inversion.*

- Let $G = (V, E)$ be a simple digraph and K the irreflexive subset of V^2. Then the inverse of G is defined as $H = (V, K \backslash E)$. Let $L^{-1} = \{H \mid \exists G \in L$ such that H is the inverse of $G\}$.

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Composition

Theorem

MGPs can be sequentially composed.

- *The first protocol must provide a stable input graph to the second protocol.*

Proof.

- Any MGP running on fixed input graphs has an equivalent MGP whose input graph may change but eventually stabilizes
  - Organize the agents into a linear graph and
  - reinitialize computation whenever the input graph changes
- The first and the second protocol composed: The first one provides a stabilizing input graph and the second one computes correctly once the input graph stabilizes

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Deciding Properties of Disconnected Graphs

- GDMPPs General Impossibility [CMS, SSS '10]:
  - No non-trivial graph language is decidable if the graph universe contains disconnected graphs
- In contrast, MGPs can exploit the complete communication infrastructure to decide properties of disconnected input graphs.

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Deciding Connectivity

Theorem

Connectivity is MGP-decidable.

Deciding Connectivity

Protocol 1 *Connectivity Protocol (CP)*

1: $Q = \{q_0, q_1, t, t', l, l'\}$, $S = \{s_0, s_1\}$,
2: $O(q_0) = 0, O(q_1) = 0, O(t) = 1, O(t') = 0, O(l) = 1, O(l') = 0$,
3: δ:

a single leader is generated
$(q_1, q_1, s_1) \to (l, t, s_1)$

the single leader turns all nodes of the input graph it can reach to followers
$(l, t, s_1) \to (t, l, s_1), (t, l, s_1) \to (l, t, s_1), (l, q_1, s_1) \to (t, l, s_1), (q_1, l, s_1) \to (l, t, s_1)$

the single leader turns all nodes that do not belong to the input graph to followers of a single leader
$(l, q_0, s_0) \to (l, t, s_0)$

two single leaders meet in the same connected component of the input graph; one is turned to follower
$(l, l, s_1) \to (l, t, s_1)$

two non-adjacent single leaders meet in the same connected component of the input graph or in different connected components (in the case of disconnected input graph); they become non-unique leaders
$(l, l, s_0) \to (l', l', s_0)$

the non-unique leaders turn all the non-leaders they meet into their followers
$(l', t, s_1) \to (t', l', s_1), (t, l', s_1) \to (l', t', s_1), (l', q_1, s_1) \to (t', l', s_1), (q_1, l', s_1) \to (l', t', s_1), (l', q_0, s_0) \to (l', t', s_0), (l', t, s_0) \to (l', t', s_0)$

two leaders of any type meet in the same component; only one single leader remains
$(l', l, s_1) \to (l, t, s_1), (l, l', s_1) \to (l, t, s_1), (l', l', s_1) \to (l, t, s_1)$

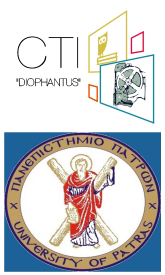
two leaders of any type meet in different components; they both become non-unique leaders
$(l', l, s_0) \to (l', l', s_0), (l, l', s_0) \to (l', l', s_0)$

a single leader restores all followers of multiple leaders to followers of single leaders
$(l, t', s_0) \to (l, t, s_0), (l, t', s_1) \to (t, l, s_1), (t', l, s_1) \to (l, t, s_1)$

Deciding Connectivity

- C-leaders: believe the graph is Connected
- D-leaders: believe the graph is Disconnected
- Followers: always adopt the output of leaders

1. All agents of the input graph are initially C-leaders and all other agents are followers.
2. 2 leaders interact via s_1-edge
   - one becomes C-leader and the other follower
3. 2 C-leaders interact via s_0-edge
   - both become D-leaders
4. Leaders always keep moving around



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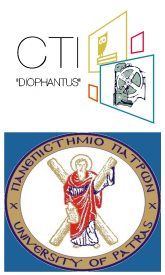

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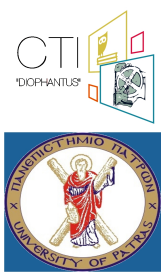

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Deciding Connectivity: Proof of Correctness

- If Connected: Eventually, the last 2 leaders interact via s_1-edge, a single C-leader remains and all followers output "connected".
- If Disconnected: Each component eventually has a single leader. Eventually, these leaders interact with each other via s_0-edges and all become D-leaders. No way to become L again. All followers output "disconnected". □

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Deciding Properties of Disconnected Graphs

- $N_{G,L}$: the # of components of G that belong to a language L
- $N_{G,\bar{L}}$: the # of components of G that do not

Theorem

Let L be a GDMPP-decidable language. Let p be a semilinear predicate on $\mathbb{N}^2$. Then $L' = \{G \mid p(N_{G,L}, N_{G,\bar{L}}) = 1\}$ is MGP-decidable.

- e.g. 25% of the components contain a 2-cycle

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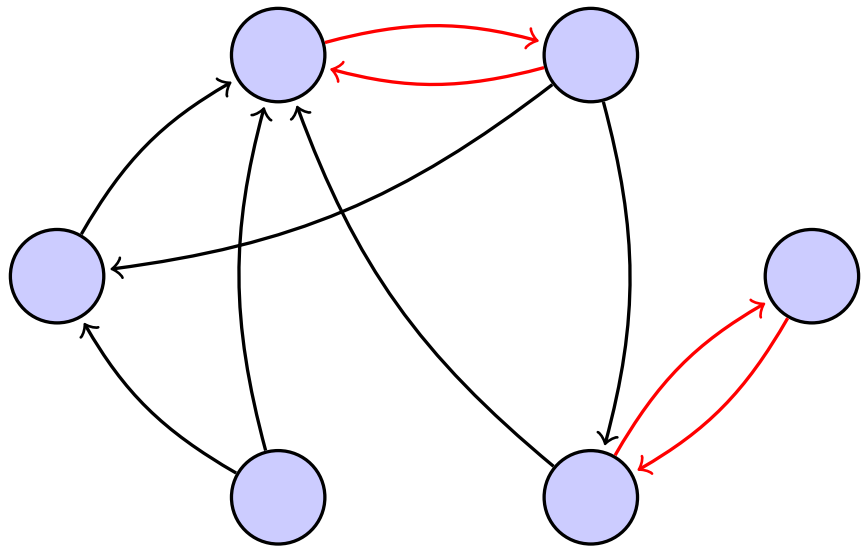
Resolving an Open Question: A GDMPP Detecting 2-cycles



- 2CYCLE $= \{G \in \mathcal{G}_{wcon} \mid G$ has at least one 2-cycle)

Theorem

2CYCLE is GDMPP-decidable.

A GDMPP: Detecting 2-cycles

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- Initially all agents are leaders and all edges are inactive
- The agents maintain a counter with the # of the active edges incident to them (at most 2)
- When 2 leaders interact only one survives (at least one leader survives forever)
- Leaders form lines and cycles by activating edges (active subgraphs)
- Any 2-cycle that becomes active cannot be deactivated
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- $G \in$ 2CYCLE
  - There is always at least one leader, which will eventually activate a 2-cycle
  - Both agents of an activated 2-cycles output 1
  - The predicate $(N_1 \geq 1)$ (there is at least 1 agent giving 1 as output) is semilinear so we can compose our protocol with a stabilizing inputs protocol computing the predicate
- $G \notin$ 2CYCLE
  - Eventually, a unique leader remains
  - Since it cannot detect a 2-cycle it forever outputs 0, while keeping moving around, and all nonleaders eventually copy this output □

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GDMPPs are Weak: An Insight

- Any GDMPP-decidable language known so far is also PP-decidable
- Focus for a moment on MPPs running on weakly-connected graphs (not necessarily complete)
- **MPU**: the corresponding class of stably computable predicates ('U' standing for "Unrestricted graphs")

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MPU = SEM.

- Any $p \in$ **MPU** is also stably computable on star graphs (1 internal node and n-1 leaves).
- The latter can be simulated by a PP with a unique leader on complete graphs
  - The leader plays the role of the internal node and stores the edge states on the leaves (which are not allowed effective communication with each other)
- Leader-PPs can be simulated by classical PPs
  - We compose a leader election protocol with a stabilizing-inputs implementation of the above PP (extended with ineffective transitions between 2 leaders)

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Remark: Leader does not seem to help PP-like models w.r.t. computational power!

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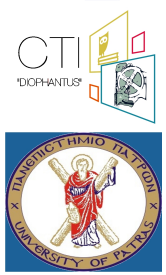

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- Gaining some insight into GDMPPs
  - They may also run on worst-case instances like star graphs
  - Star graphs are hard because the internal node has an uncountable ($O(n)$) number of neighbors (the leaves)

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MGPs are Strong: An Exact Characterization

Theorem

GMGP *is equal to the class of all decidable graph languages by a NTM of linear space which receives the input graph by its adjacency matrix representation.*

Proof.

- Construct a spanning pseudo-path subgraph
- Visit one after the other all distributed cells (from left to right)
- Keep count of the distance of the corresponding nodes from the left endpoint
- Store each s_1 edge (u, v) in the leftmost cells of the memory by putting a 1 in the adjacency matrix position (u, v)
- Then simply simulate the NTM on the constructed input
- In the worst case, the adjacency matrix is of the same order as the interaction graph and needs $O(n^2)$ space, which is available

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Conclusions

- Investigated MPPs ability to decide graph properties of input graphs over a complete communication infrastructure
- Some Properties: closure, sequential composition
- Some Protocols: connectivity, cardinalities of components
- Exact Characterization: Equivalent to linear-space NTM
- Resolved open questions: GDMPPs can detect 2-cycles, MPPs are semilinear on unrestricted graphs, leaders do not enhance PPs
- Almost resolved: GDMPPs are weak (probably semilinear)

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FRONTS

- This work has been partially supported by the ICT Programme of the European Union under contract number ICT-2008-215270 (FRONTS).
- FRONTS is a joint effort of eleven academic and research institutes in foundational algorithmic research in Europe.

- The effort is towards establishing the foundations of adaptive networked societies of tiny artefacts.

Thank You!

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