# Expansion Properties of Large Social Graphs

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**Abstract.** Social network analysis has become an extremely popular research area, where the main focus is the understanding of networks' structure. In this paper, we study the expansibility of large social graphs, a structural property based on the notion of expander graphs (i.e. sparse graphs with strong connectivity properties). It is widely believed that social networks have poor expansion properties, due to their community-based organization. Moreover, this was experimentally confirmed on small scale networks and it is considered as a global property of social networks (independent of the graph's size) in many applications. What really happens in large scale social graphs? To address this question, we measure the expansion properties of several large scale social graphs using the measure of subgraph centrality. Our findings show a clear difference on the expansibility between small and large scale social networks, and thus structural differences. Our observations could be utilized in a range of applications which are based on social graphs' structure.

**Keywords:** Social networks, Expansion, Measurement, Graph Mining

## 1 Introduction

Recently, there has been a lot of interest in the study of complex network structures arising in many diverse settings. Characteristic examples are networks from the domain of sociology (e.g. social networks), technological and information networks (e.g. the Internet, the Web, e-mail exchange networks, social interaction networks over social media applications), biological networks (e.g. protein interactions), collaboration and citation networks (e.g. co-authorship networks), and many more [20]. The research interest has mainly focused on understanding the structure, the organization, and the evolution of these networks, and many interesting results have been produced [2].

A better and deeper understanding of network's structure could have multiple benefits in several domains (e.g. better design for graph algorithms and applications). Towards this direction, in this work we study the *expansibility* of large social graphs, a structural property based on concepts from the theory of expander graphs [10]. Our main goal is to explore the expansion properties

**Table 1.** Symbols and definitions.

| Symbol | Definition |
|---|---|
| $G$ | Graph representation of datasets |
| $V,\ E$ | Set of nodes and edges for graph $G$ |
| $\|V\|,\ \ \|E\|$ | Number of nodes and edges |
| $N(S)$ | Neighborhood nodes of node set $S$ |
| $h(G)$ | Expansion factor of graph $G$ |
| | (or isoperimetric number or Cheeger constant) |
| $\mathbf{A}$ | Adjacency matrix of a graph |
| $a_{ij}$ | Entry in matrix $\mathbf{A}$ |
| $\lambda_i$ | $i$-th largest eigenvalue |
| $u_{ij}$ | $i$-th component of $j$-th eigenvector |
| $SC(i)$ | Subgraph centrality of node $i$ |

of large scale social networks and compare it with known results from previous studies on small graphs, in order to extract useful conclusions about social networks' structure. Table 1 gives a list of used symbols with their definitions.

### Expansion and Expander Graphs

Informally, a graph is a good expander if it is simultaneously sparse and highly connected. More precicely, given a graph $G = (V, E)$, the expansion of any subset of nodes $S \subset V$ with size at most $\frac{|V|}{2}$, is defined as the number of its neighborhood nodes (i.e. those nodes who have one endpoint inside $S$ and the other outside) over the size of the subset $S$. That is, if $N(S)$ are the neighborhood nodes of $S$, the expansion factor of the set $S$ is $\frac{|N(S)|}{|S|}$. A graph is considered to have *good expansion properties* if every such subset of nodes has expansion at least $h(G)$, i.e. $h(G) \geq \frac{|N(S)|}{|S|}, \forall S \subset V$ and $|S| < \frac{|V|}{2}$. In other words, the expansion factor of a graph is defined as the minimum expansion over all subsets [10]:

$$h(G) = \min_{\left\{S:|S|\leq \frac{|N|}{2}\right\}} \frac{|N(S)|}{|S|}. \tag{1}$$

Expansion properties can offer crucial insights into the structure of a graph, and in particular they can inform us about the presence or not of edges which can act as bottlenecks inside the network. This practicaly means that measuring the expansibility of a graph we are able to know to what extent the graph has a modular structure or not. Large expansion factor implies good expansion properties, which means that any subset of nodes will have a relatively large number of edges with one endpoint in this set, and thus poor modularity. In other words, if we think these subsets as cuts of a graph, good expansibility require cuts with large size (i.e. large number of edges crossing the cut). On the other hand, bad expansibility is the opposite behavior. For any subset of nodes it is impossible to satisfy the constraint for a large neighborhood. Hence, such kind of graphs can be easily separated into disconnected subgraphs with the elimination of a small number of edges. It is clear that using notions from the field of sociology and social networks, graphs with poor expansibility correspond to graphs with good community structure.

**Contributions and Summary of our Results**

In this paper we measure the expansibility of several large social graphs. Based on the above discussion, we expect that social networks will exhibit bad expansion properties, because of the fact that they are organized in communities, i.e. groups of nodes with high density of edges within them, and much lower density between different groups [21]. This structural property was confirmed experimentally from previous studies [7] on *small* social networks, and in several cases is considered as a *global property* of social networks, *independent* of the graph size (e.g., some generative models for social networks are trying to generate synthetic social graphs satisfying this bad expansion property). However, does the same result apply to *large scale* social networks? In other words, how different is the structure of social graphs with a large number of nodes and edges, if any, from that of small graphs?

This is the main question we are trying to answer in this work. We measure the expansion properties of several social graphs with a large number of nodes and edges. In order to do this, we consider the fact that graphs with good expansion properties exhibit large spectral gap between the two largest eigenvalues of the adjacency matrix [10]. Then, utilizing this property together with the measure of subgraph centralilty [6], [7], we characterize the expansibility of these social graphs. Our findings suggest that *large scale social networks, in contrast to small ones, show good expansibility.* This point is particularly significant since it can help us towards a better understanding of large social networks' structure. Furthermore, these observations can be exploited in several domains such as structure-based classification schemes for networks, searching in networks [16] and in applications which may require robustness of the social network over social media applications.

## 2 Related Work

In this section we review the related work, which can be placed into three main categories: graph structure, applications and spectral graph analysis.

**Graph Structure.** There is a vast literature on methods for understanding the structure of social networks [21], [11], [18], [15] and generally of complex networks [20]. The key step for these methods is finding properties and laws which the graphs obey. Studying static snapsots of graphs has led to the discovery of interesting properties such as the *power law degree distribution* [9], the *small diameter* [1] and the *triangle power law* [25]. Futhermore, Leskovec et al. [13], [14] showed that time-evolving graphs have diameter which shrinks and stabilizes over time and obey the *densification power law*. For a nice survey one can consult the recent work of Chakrabarti, Faloutsos, and McGlohon [2]. Estrada [7] studied expansion properties of complex networks and showed that social graphs exhibit poor expansibility. However, in contrast with our work, it focuses on small scale networks. On the other hand, we explore large scale social networks and our results suggest a clear difference between their structure (in terms of expansibility) with that of small social graphs.

**Applications.** The understanding of a network's structure can be exploited in several domains and applications. Generating realistic graphs [2] is such an application, where generators should satisfy the observed properties. Other domains are searching in networks [16], sampling [17] and rumor spreading [4].

**Spectral Graph Analysis.** Analyzing graphs using spectral techniques (i.e. the eigenvalues and eigenvectors of a matrix representation of the graph (mainly adjacency and Laplacian matrices)) has a long history [3]. More recent related works include spectral algorithms for community detection [23] and spectral counting of triangles in large graphs [25]. As we will see next in this paper, the measure which is used for characterizing the expansibility of social graphs (subgraph centrality) can be computed using the spectrum of the adjacency matrix of the graph.

## 3 Measuring Expansion Properties

In this section we present the method we used for measuring the expansion properties of social networks, to characterize them as networks with "good" or "bad" expansibility. As we state previously, in order to compute the expansion factor of a graph (which fully characterizes its expansion properties), we need to compute the minimum fraction of neighborhood nodes, over the nodes inside the subset, for all possible subsets of nodes with size at most $\frac{|V|}{2}$. Since this is an NP-hard problem [19], and thus intractable to compute, we need approximation techiniques for the expansion factor of a graph.

Thanks to a very well known result in the field of spectral graph theory, the expansion factor can be approximated using the spectrum of the adjacency matrix $\mathbf{A}$ of the graph, and more precicely the difference between the largest and second largest eigenvalues of $\mathbf{A}$. This difference $\Delta\lambda = \lambda_1 - \lambda_2$ is known as the *spectral gap* of matrix $\mathbf{A}$ and it is related to the expansion factor $h(G)$ through the Alon - Milman inequality[3],

$$\frac{\Delta\lambda}{2} \leq h(G) \leq \sqrt{2\lambda_1 \Delta\lambda}. \quad (2)$$

[3] This is also known as Cheeger inequality.

Large spectral gap implies big expansion factor and thus a graph with good expansion properties. On the other hand, if $\lambda_2$ is close enough to $\lambda_1$, the spectral gap will be small and the graph will show poor expansibility.

The above discussion suggests a simple way for characterizing the expansion properties of a graph: compute the spectral gap and if this is large, the graph should have good expansion properties. However, a crucial question in the above claim is *how large* the spectral gap should be for a graph to have good expansibility. As we will see from the experimental study in real-world networks, it is very difficult to measure the quantity of interest solely from the spectral gap of the adjacency matrix.

In this paper we measure the expansibility of a graph using the notion of *subgraph centrality* [6], employing a solution proposed by Estrada [7]. The reason for this decision is twofold: first of all, as we will see in the rest of this paper, the method based on subgraph centrality provides a clear distinction between graphs with different expansion properties. The second reason is that using this method, we can easily compare our results with that of [7], trying to find differences between the structure of large and small scale social graphs.

### 3.1 Subgraph Centrality

In this section we present the subgraph centrality measure [6] which is the basis for the estimation of the expansion character of a graph. Like other centrality measures in the field of graph theory and network analysis (e.g., degree centrality, betweenness centrality), subgraph centrality determines the importance of a node in the graph taking into consideration all the subgraphs in which the node participates.

More precicely, the subgraph centrality $SC(i)$ of a node $i \in N$ is calculated based on the total number of closed walks in a graph. A closed walk of specific length $\ell$ is an alternating sequence of nodes and edges starting and ending with a node, $v_1, e_1, v_2, e_2, \ldots, e_{\ell-1}, v_\ell$, where $e_i = (v_i, v_{i+1}) \in E, \forall i = 1, \ldots, \ell-1$ and $v_1 = v_\ell$. For instance, a closed walk of length three represents a triangle. The subgraph centrality of a node $i$ is defined as the sum of closed walks with different lengths, starting and ending at node $i$. However, all these walks with different lengths do not contribute equally to the centrality of the node; shorter walks contribute more (this happens because of the fact that in real-world graphs small subgraphs tend to be more interesting (e.g., triangles)). Thus, the subgraph centrality of node $i$ is given by

$$SC(i) = \sum_{\ell=0}^{\infty} \frac{\mathbf{A}_{ii}^{\ell}}{\ell!}, \tag{3}$$

where the diagonal entry $\alpha_{ii}$ of the matrix $\mathbf{A}^\ell$ contains the number of walks of length $\ell$ that begin and end at the same node $i$. Using techniques from spectral graph theory, it can be proved that the subgraph centrality can be obtained from the spectrum of the adjacency matrix $\mathbf{A}$ of the graph. Because of the fact that (3) counts both even and odd length closed walks and more precicely even length walks may be trivial (moving forth and back in the graph), we keep only odd length walks[4] [6]:

$$SC(i) = \sum_{j=1}^{N} u_{ij}^2 \sinh(\lambda_j). \tag{4}$$

Now we can write (4) in the form

$$SC(i) = u_{i1}^2 \sinh(\lambda_1) + \sum_{j=2}^{N} u_{ij}^2 \sinh(\lambda_j), \tag{5}$$

where $u_{i1}$ is the $i$-th component of the principal eigenvector (eigenvector corresponding to the largest eigenvalue $\lambda_1$). If the graph has good expansion properties, which means that $\lambda_1 \gg \lambda_2$, then $u_{i1}^2 \sinh(\lambda_1) \gg \sum_{j=2}^{N} u_{ij}^2 \sinh(\lambda_j)$ and relation (5) could be written as

$$SC(i) \approx u_{i1}^2 \sinh(\lambda_1). \tag{6}$$

This means that the principal eigenvector $u_{i1}$ is related to $SC(i)$ as

$$u_{i1} \propto \sinh^{-1/2}(\lambda_1)\ SC(i)^{1/2}. \tag{7}$$

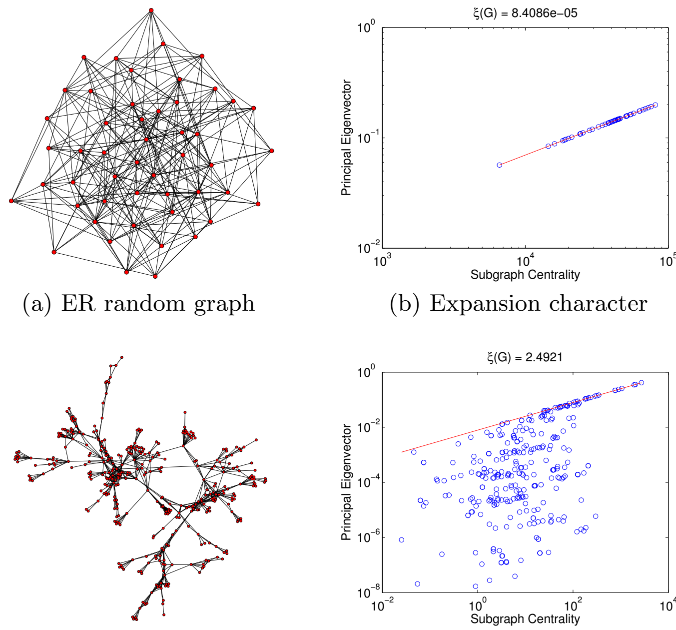
This relation suggests that if the graph has good expansion properties (big spectral gap), $u_{i1}$ will be proportional to $SC(i)$ and a log-log plot of $u_{i1}$ vs. $SC(i)$, $\forall i \in N$ will show a linear fit with slope 1/2 [7]. Thus, good expansion implies a power-law relationship between the principal eigenvector and the subgraph centrality. On the other hand, graphs with poor expansibility will deviate from this property. Moreover, this behavior can be summarized in the quantity $\xi(G)$, which captures exactly the expansion character of a graph [8]:

$$\xi(G) = \sqrt{\frac{1}{|N|} \sum_{i=1}^{|N|} \Big\{ \log(u_{i1}) - \Big( \log A + \frac{1}{2} \log(SC(i)) \Big) \Big\}^2}, \tag{8}$$

where $A = \sinh^{-1/2}(\lambda_1)$. This quantity measures the deviation from the "perfect" linear correlation (in log scale), which occurs when the spectral gap $\lambda_1 - \lambda_2$ is large (and thus the graph has good expansion properties). This is exactly what we propose to use in this paper for measuring the expansion properties of real-world social graphs.

For a better understanding and illustration, we apply this method to two graphs with known expansion properties. The first one is a random graph with 50 nodes produced by the Erdös-Rényi (ER) random graph model [5] with probability $p = 0.3$ (Fig. 1 (a)) and the second one is Newman's collaboration network between 379 researchers in the area of network science (Fig. 1 (c)) [22].

[4] The graphs used in this study are non-bipartite and thus the number of closed walks of odd length is different from zero.




(a) ER random graph (b) Expansion character

(c) Network science graph (d) Expansion character

**Fig. 1.** Two graphs with known expansion properties and the plots of the principal eigenvector vs. subgraph centrality in log-log scale.

Random graphs are known to have good expansibility [10], and thus we expect linear correlation in log-log scales between the principal eigenvector and subgraph centrality. On the other hand, Newman's collaboration network has bad expansion character because of the fact that nodes form dense modules, with sparse connections between different modules. Hence, we expect deviation from this "perfect" linear correlation. Figure 1 (b) and (d) depicts these results. Also, we can observe that $\xi(G)$ is much smaller for the ER graph compared with the second one, which agrees with the above discussion.

## 4 Experimental Results

Equipped with the tools presented in Section 3, we measure the expansion properties of different real-world social graphs shown in Table 2. All these graphs represent social networks with a large number of nodes and edges. The selection of these datasets, except from their large scale, is based on the fact that they were formed under different "rules" and conditions. On the one hand we have networks where edge creation is based on mutual knowledge between individuals (e.g., co-authorship networks). On the other hand, there is a set of social networks, some of which are formed over social media applications, that may not require mutual knowledge (and sometimes confirmation from the other side) for

**Table 2.** Summary of real-world networks used in this study.

| Network | Nodes | Edges | Description |
|---|---|---|---|
| Epinions [24] | 75,877 | 405,739 | Who trusts whom network |
| Email-EUAll [14] | 224,832 | 340,795 | Email network |
| Slashdot [15] | 77,360 | 546,487 | Slashdot social network (Nov. '08) |
| Wiki-Vote [12] | 7,066 | 100,736 | Wikipedia who-votes-on-whom network |
| Facebook [26] | 63,392 | 816,886 | Facebook New Orleans social network |
| Youtube [18] | 1,134,890 | 2,987,624 | Social network from Youtube site |
| CA-astro-ph [14] | 17,903 | 197,031 | Co-authorship network in Astro Physics |
| CA-gr-qc [14] | 4,158 | 13,428 | Co-author. network in General Relativity |
| CA-hep-th [14] | 8,638 | 24,827 | Co-author. network in High Energy Phys. |

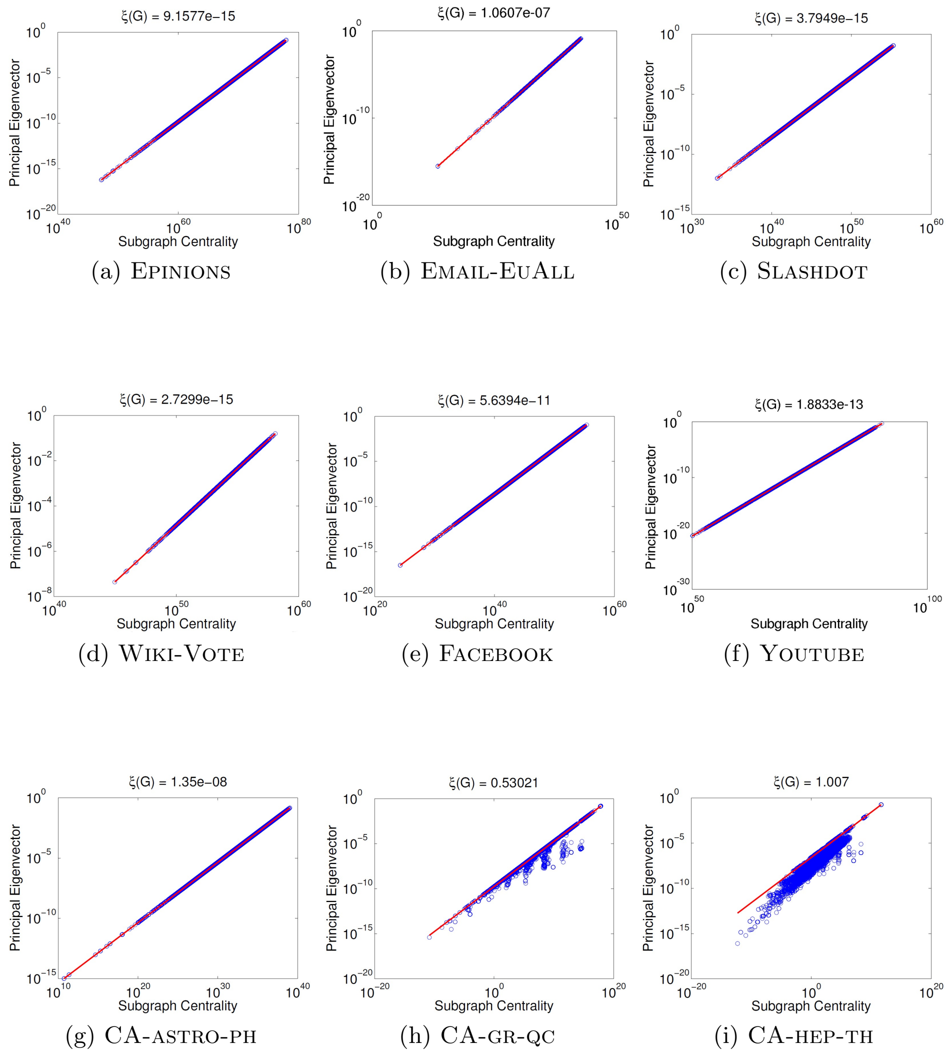
the interaction (e.g. Youtube). In all cases, we consider the graphs as unweighted and undirected. Moreover, we extract the largest connected component and use it as a good representative of the whole graph (this is a standard approach in such kind of studies).

Figure 2 presents plots of the expansion character of the graphs we examined, together with the values $\xi(G)$. From a first look, it is clear that almost all social graphs (except the last two which we will discuss later) exhibit good expansion properties, showing linear correlation between the principal eigenvector and subgraph centrality in log-log scales. In all plots we have included a red line which represents this ideal behavior in case of graphs with big spectral gap and therefore good expansibility.

The results suggest that social graphs depicted in Fig. 2 (a)-(g), expand very well allowing the selection of arbitrary subsets of nodes with size at most $\frac{|V|}{2}$, such that for every set there is a relatively large number of edges with one endpoint inside the set and the other outside (in other words, every selection of such a subset creates a cut in the graph with a relative large size). Thus, a first conclusion is that these social graphs lack of edges which can act as bottlenecks. Furthermore, this result implies that the nodes inside the networks we examined are not organized based on a clear modular architecture. More precicely, a basic characteristic of the networks' structure is the absence of well defined clusters which can be easily seperated from the whole graph. In other words, the networks lack of clusters (communites) with a clear difference between the number of intra-cluster edges and inter-cluster edges[5].

However, in what degree are the above observations expected? Before trying to answer this question we must repeat that the used datasets correspond to social networks and on-line social networks from social media applications, with a large number of nodes and edges. It is known that the organization of social networks is based on communities (i.e. subgraphs with high intra-community

[5] We must note here that our findings do not imply the absence of communities from social graphs, but the subgraphs which may correspond to communities cannot be easily isolated, since they have a relatively large number of extra-community connections.




(a) Epinions (b) Email-EuAll (c) Slashdot

(d) Wiki-Vote (e) Facebook (f) Youtube

(g) CA-astro-ph (h) CA-gr-qc (i) CA-hep-th

**Fig. 2.** Expansion properties of large social networks presented in Table 2. All plots depict the principal eigenvector vs. the subgraph centrality in log-log scale, and the $\xi(G)$ value for each graph.

and low inter-community edges). As a result, we expect that social networks will have poor expansion character because of the presence of communities. This means that it is very difficult for all the subsets of nodes to satisfy the constraint for good expansibility. In [7], the author measured the expansibility of a large number of real-world social networks, and showed that almost all of them have bad expansion properties, which is intuitively expected from the above discussion. However, it is very important to note that the social graphs studied in [7], have a small number of nodes and edges. Moreover, none of them has arisen from social media applications and generally online social networking, but almost all formed by physical interaction between people.

On the other hand, as our results suggest, the expansion character of large scale social graphs is completely different from that of small scale networks. Almost all studied social networks exhibit good expansibility, which we consider that is mainly due to two reasons. The first one, and the most obvious, is the scale of the network. It seems that, in large scale social graphs it is difficult to find subsets of nodes which can be easily isolated. For example, consider the co-authorship networks CA-ASTRO-PH, CA-GR-QC and CA-HEP-TH. While these networks are formed in a similar way (collaboration between scientists), the first one has about $18K$ nodes and $200K$ edges, while the other two have much smaller order (number of nodes) and size (number of edges) ($4K$ nodes, $13K$ edges and $8K$ nodes, $25K$ edges respectively). Figures 2 (g), (h) and (i) show the expansion properties of these graphs. We can observe their different behavior, where the larger one shows good expansibility with a very small $\xi(G)$ value ($1.35 \times 10^{-8}$), while the other two do not show this property ($\xi(G) = 0.53021$ and $\xi(G) = 1.007$ respectively).

The second reason we consider for justifying these findings is that most of these networks are created over social networking and social media applications. Thus, because of the fact that the interaction may not require knowledge from both parts, it is easier to be achieved. Of course, something like that is very difficult to happen in social networks which require knowledge of the other part for an interaction.

**Application Example.** How these findings could be utilized in a real application, such as decentralized search in complex networks? This is a common problem in many applications, where starting from one initial node, we must locate a target node inside the network, without full knowledge of the global network structure (topology). Since the computation of the shortest path to the target node is unable, a strategy is to visit nodes using only local information, in such a way that every subset of visiting nodes has a large neighborhood and thus good expansibility (the goal is to reach the target node, minimizing the number of required steps). Since our findings suggest that large scale social graphs exhibit good expansion properties, the networks tend to be more searchable, making the above searching strategy more efficient.

**Computational Issues.** While subgraph centrality (4) provides a powerfull tool for measuring the expansion properties of a graph, it requires the computation of all eigenvalue - eigenvector pairs $(\lambda_i, \mathbf{u}_i),\ \forall i \in N$, of the adjacency matrix $\mathbf{A}$. While this may not be a problem for small graphs, it becomes a computational bottleneck for large scale networks. In order to overcome this, we use the observation of [25], which states that the eigenvalues of real-world graphs are almost symmetric around zero, meaning that their signs tend to alternate. Moreover, because of the fact that the $\sinh(\cdot)$ function keeps the sign of the eigenvalues, we can use only the top strongest eigenvalues and their corresponding eigenvectors to achieve an excellent approximation of the subgraph centrality (in our experiments we keep the first 30 strongest pairs).

## 5 Conclusions

In this paper we measured the expansion properties of several large scale social graphs, using the measure of subgraph centrality. Our findings show that, in contrast to small social networks, large scale social graphs generally exhibit good expansibility. This is something that has not appreciated previously, and in many cases social graphs were characterized as graphs with poor expansion properties, indepedent of their size. Our observations, except for a better understanding of social networks' structure, could be possibly utilized in several domains and applications such as searching in networks, community discovery and more generally in applications over social networks where the robustness of the underlying structure is a crucial factor. In future work, we plan to further investigate and understand the underlying mechanisms which cause this behavior and how these findings could affect applications which consider the structure of social networks.

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