Power Consumption Problems in Ad-Hoc Wireless Networks*

Ioannis Caragiannis, Christos Kaklamanis, and Panagiotis Kanellopoulos

Computer Technology Institute and
Dept. of Computer Engineering and Informatics
University of Patras, 26500 Rio, Greece

Wireless networks have received significant attention during the recent years. Especially, *ad hoc wireless networks* emerged due to their potential applications in battlefield, emergency disaster relief, etc. [13]. Unlike traditional wired networks or cellular wireless networks, no wired backbone infrastructure is installed for ad hoc wireless networks.

A node (or station) in these networks is equipped with an omnidirectional antenna which is responsible for sending and receiving signals. Communication is established by assigning to each station a transmitting power. In the most common power attenuation model [13], the signal power falls as $1/r^{\alpha}$, where r is the distance from the transmitter and α is a constant which depends on the wireless environment (typical values of α are between 1 and 6). So, a transmitter can send a signal to a receiver if $\frac{P_s}{d(s,t)^{\alpha}} \geq \gamma$ where P_s is the power of the transmitting signal, $d(s,t)$ is the Euclidean distance between the transmitter and the receiver, and γ is the receiver's power threshold for signal detection which is usually normalized to 1. So, communication from a node s to another node t may be established either directly if the two nodes are close enough and s uses adequate transmitting power, or by using intermediate nodes. Observe that due to the nonlinear power attenuation, relaying the signal between intermediate nodes may result in smaller power consumption.

A crucial issue in ad hoc wireless networks is to support communication patterns that are typical in traditional networks. These include broadcasting, multicasting, and gossiping (all–to–all communication). Since establishing a communication pattern strongly depends on the power levels, the important engineering question to be solved is to guarantee a desired communication pattern minimizing the total power consumption. In this work, we consider a series of power consumption problems which we formulate below.

Consider a complete directed graph $G = (V, E)$, where $|V| = n$, with a non-negative edge cost function $c : E \rightarrow R^+$. Given a non-negative node weight assignment $w : V \rightarrow R^+$, the *transmission graph* G_w is the directed graph defined as follows. It has the same set of nodes as G and a directed edge (u, v) belongs to G_w if the weight assigned to node u is at least the cost of the edge (u, v), i.e., $w(u) \geq c(u, v)$. Intuitively, the weight assignment corresponds to the

* This work was partially supported by the European Union under IST FET Project ALCOM–FT, IST FET Project CRESCCO, and RTN Project ARACNE.

R. Solis-Oba and K. Jansen (Eds.): WAOA 2003, LNCS 2909, pp. 252–255, 2004.

power levels at which each node operates (i.e., transmits messages) while the cost between two nodes indicates the minimum power required to send messages from one node to the other. Usually, the edge cost function is symmetric (i.e., $c(u,v) = c(v,u)$). In ideal cases, we can assume that the nodes of the graph are points in a Euclidean space and the cost of an edge between two nodes is the Euclidean distance of the corresponding points raised to a constant power. Asymmetric edge cost functions can be used to model medium abnormalities or batteries with different power levels [12].

The problems we study can be generally stated as follows. Given a complete directed graph $G = (V, E)$, where $|V| = n$, with non-negative edge costs $c : E \rightarrow R^+$, find a non-negative node weight assignment $w : V \rightarrow R^+$ such that the transmission graph G_w maintains a connectivity property and the sum of weights is minimized. Such a property is defined by a requirement matrix $R = (r_{ij}) \in \{0, 1\}$ where r_{ij} is the number of directed paths required in the transmission graph from node v_i to node v_j.

In Minimum Energy Multicast Tree (MEMT), the connectivity property is defined by a root node v_0 and a set of nodes $D \subseteq V - \{v_0\}$ such that $r_{ij} = 1$ if $i = 0$ and $v_j \in D$ and $r_{ij} = 0$, otherwise. The Minimum Energy Broadcast Tree (MEBT) is the special case of MEMT with $D = V - \{v_0\}$. Liang [12] presents an intuitive reduction of any instance of MEMT to an instance of Directed Steiner Tree (DST). An $O(|D|^\epsilon)$-approximation algorithm for any $\epsilon > 0$ then follows by using an approximation algorithm for DST[1]. The authors show in [4] that MEMT is also at least as hard as DST (this is also observed in [2]). Using a recent inapproximability result for DST, we obtain an $O(\ln^{2-\epsilon} n)$ inapproximability result for MEMT. Hence, improvements on the approximability bounds of MEMT strongly depend on new results for DST. It is strongly believed that polylogarithmic approximation algorithms for the latter do exist. In symmetric graphs, there exists an $O(\ln |D|)$-approximation algorithm. This is shown in [4] where instances of MEMT in symmetric graphs are reduced to instances of Node-Weighted Steiner Tree, a problem which is known to be approximable within a logarithmic factor. Note that this result is asymptotically optimal since it can be easily seen that MEMT in symmetric graphs is at least as hard to approximate as Set Cover [5]. MEBT is easier and can be approximated within $O(\ln n)$ as it was recently shown in [4] and [2]. In [4], the authors show that the corresponding instance of DST has some very good properties and can be approximated within a logarithmic factor. Calinescu et al. [2] achieve a similar approximation bound by a simpler algorithm which constructs a tree incrementally. Sophisticated set-covering techniques are used for the analysis. Constant approximation algorithms for the special case of MEBT where the nodes of the input graph correspond to points in the Euclidean plane are presented in [5, 14]. The special case of MEBT where the nodes are points in a line can be solved in polynomial time [3, 6].

---

[1] Due to lack of space, references to known results for combinatorial problems which are crucial in the proofs of the results presented have been omitted from this survey. The interested reader may see the papers cited here and the references therein.

In Minimum Energy Steiner Subgraph (MESS), the requirement matrix is symmetric. Alternatively, we may define the problem by a set of nodes $D \subseteq V$ partitioned into p disjoint subsets $D_1, D_2, ..., D_p$. The entries of the requirement matrix are now defined as $r_{ij} = 1$ if $v_i, v_j \in D_k$ for some k and $r_{ij} = 0$, otherwise. The Minimum Energy Subset Strongly Connected Subgraph (MESSCS) is the special case of MESS with $p = 1$ while the Minimum Energy Strongly Connected Subgraph (MESCS) is the special case of MESSCS with $D = V$ (i.e., the transmission graph is required to span all nodes of V and to be strongly connected). Although the approximability of MESS is yet unknown, MESSCS has the same approximability properties as MEMT, i.e., it can be approximated within $O(|D|^\epsilon)$ but cannot be approximated within $O(\ln^{2-\epsilon} n)$. Whether there are polylogarithmic approximations for this problem is an interesting open problem and strongly depends on new results for DST. MESCS can be approximated within a logarithmic factor using an algorithm for MEBT as a subroutine. This result is asymptotically optimal. In symmetric graphs, MESS, MESSCS and MESCS have constant approximation algorithms using algorithms for network design problems as a subroutine [4] (see also [11]). The respective bounds are a 2-approximation algorithm for MESCS using optimal Spanning Tree solutions (this was first proved by Kirousis et al. in [10]), a 2ρ-approximation algorithm for MESSCS using ρ-approximate Steiner Tree solutions, and a 4-approximation algorithm for MESS using a 2-approximation algorithm for Steiner Forest. Approximation schemes are unlikely to exist for these problems in symmetric graphs since inapproximability bounds have been proved for MESCS [8] using approximation-preserving reductions from Vertex Cover in bounded-degree graphs. The special case of MESCS where the nodes are points in a line can be solved in polynomial time [7].

The authors of [1] study MESCS under the extra requirement that the transmission graph contains a bidirected subgraph which maintains the connectivity requirements of MESCS. By adding this extra requirement to MESS and MESSCS, we obtain the bidirected MESS and bidirected MESSCS, respectively. In [4], it is shown that instances of bidirected MESS can be reduced to instances of Node-Weighted Steiner Forest, a problem for which logarithmic approximations exist. As corollaries, we obtain $O(\ln |D|)$- and $O(\ln n)$-approximation algorithms for bidirected MESSCS and bidirected MESCS, respectively. A slightly inferior logarithmic approximation bound for bidirected MESCS is presented in [2] using different techniques. These results asymptotically match the inapproximability result for bidirected MESCS of Althaus et al. [1]. For symmetric graphs, the positive and negative results for MESS, MESSCS, and MESCS hold for their bidirected versions as well. Better approximation algorithms for bidirected MESCS are presented in [1].

We have surveyed results on power consumption problems where the connectivity requirements are defined by 0-1 requirement matrices. A natural extension is to consider matrices with non-negative integer entries r_{ij} denoting that at least r_{ij} node-disjoint paths are required from node v_i to node v_j. This extension leads to interesting power consumption problems for fault torelant wireless communication. Some recent results on such problems can be found in [9, 11].

References

1. E. Althaus, G. Călinescu, I. Măndoiu, S. Prasad, N. Tchervenski, and A. Zelikovsky. Power Efficient Range Assignment in Ad-Hoc Wireless Networks. In *Proc. of the IEEE Wireless Communications and Networking Conference (WCNC '03)*, IEEE Computer Society Press, pp. 1889-1894, 2003.
2. G. Călinescu, S. Kapoor, A. Olshevsky and A. Zelikovsky. Network Lifetime and Power Assignment in Ad-Hoc Wireless Networks. In *Proc. of the 11th Annual European Symposium on Algorithms (ESA '03)*, LNCS 2832, Springer, 2003.
3. I. Caragiannis, C. Kaklamanis and P. Kanellopoulos. New Results for Energy-Efficient Broadcasting in Wireless Networks. *In Proc. of the 13th Annual International Symposium on Algorithms and Computation (ISAAC '02)*, LNCS 2518, Springer, pp. 332-343, 2002.
4. I. Caragiannis, C. Kaklamanis and P. Kanellopoulos. Energy-Efficient Wireless Network Design. *In Proc. of the 14th Annual International Symposium on Algorithms and Computation (ISAAC '03)*, 2003, to appear.
5. A. E. F. Clementi, P. Crescenzi, P. Penna, G. Rossi, and P. Vocca. On the Complexity of Computing Minimum Energy Consumption Broadcast Subgraphs. In *Proc. of the 18th Annual Symposium on Theoretical Aspects of Computer Science (STACS '01)*, LNCS 2010, Springer, pp. 121–131, 2001.
6. A. E. F. Clementi, M. Di Ianni, R. Silvestri. The Minimum Broadcast Range Assignment Problem on Linear Multi-Hop Wireless Networks. *Theoretical Computer Science*, 1-3(299): 751-761, 2003.
7. A. E. F. Clementi, P. Penna, A. Ferreira, S. Perennes, and R. Silvestri. The Minimum Range Assignment Problem on Linear Radio Networks. *Algorithmica*, 35(2): 95-110, 2003.
8. A. E. F. Clementi, P. Penna, and R. Silvestri. Hardness Results for the Power Range Assignment Problem in Packet Radio Networks. In *Proc. of Randomization, Approximation, and Combinatorial Optimization (RANDOM/APPROX '99)*, LNCS 1671, Springer, pp. 197–208, 1999.
9. M. Hajiaghayi, N. Immorlica and V. Mirrokni. Power Optimization in Fault-Tolerant Topology Control Algorithms for Wireless Multi-hop Networks. In *Proc. of the 9th Annual International Conference on Mobile Computing and Networking (MOBICOM '03)*, pp. 300-312, 2003.
10. L. M. Kirousis, E. Kranakis, D. Krizanc, and A. Pelc. Power Consumption in Packet Radio Networks. *Theoretical Computer Science*, 243(1-2):289-305, 2000.
11. E. Lloyd, R. Liu, M. Marathe, R. Ramanathan and S.S. Ravi. Algorithmic Aspects of Topology Control Problems for Ad hoc Networks. In *Proc. of 3rd ACM International Symposium on Mobile Ad Hoc Networking and Computing (MOBIHOC '02)*, pp. 123-134, 2002.
12. W. Liang. Constructing Minimum-Energy Broadcast Trees in Wireless Ad Hoc Networks. In *Proc. of 3rd ACM International Symposium on Mobile Ad Hoc Networking and Computing (MOBIHOC '02)*, pp. 112-122, 2002.
13. T. S. Rappaport. Wireless Communications: Principles and Practices. *Prentice Hall*, 1996.
14. P.-J. Wan, G. Călinescu, X.-Y. Li, and O. Frieder. Minimum-Energy Broadcasting in Static Ad Hoc Wireless Networks. *Wireless Networks*, 8(6):607-617, 2002.